

Excitation of Leaky Modes on Multilayer Stripline Structures

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Abstract—A quasi-analytical method for calculating the excitation of leaky modes on multilayer stripline structures by a finite source is presented in this paper. Simple sources such as an infinitesimal dipole near the conducting strip or a delta-gap feed on the conducting strip of the transmission line are considered. The method uses a numerically constructed Green's function for the source in the presence of the conducting strip, which is calculated from Fourier transform theory in terms of a one-dimensional Green's function for a line source in the presence of the conducting strip. The numerical Green's function involves a one-dimensional integration in the longitudinal wavenumber plane. The residue contributions from the poles of the Green's function define the excitation amplitudes of the leaky and bound modes that exist on the structure. The numerical Green's function is also used to numerically calculate the complete current on the strip excited by the source. The correlation between the leaky-mode current and the complete current is used to define the extent of the physical meaning of the leaky mode. The generalized pencil of functions (GPOF) method is used to study this correlation by resolving the complete current on the strip into exponential waves, which are then compared with the current of the leaky mode. The physical meaning of the leaky modes is also analytically examined by consideration of the branch cuts in the longitudinal wavenumber plane for the numerical Green's function integration. A "path consistency condition" is established as a necessary condition for the physical meaning of the leaky mode.

Index Terms—Leaky waves, microwave integrated circuits, planar transmission lines, planar waveguides, stripline, transmission lines, waveguide excitation.

I. INTRODUCTION

THE existence of leaky modes on printed-circuit transmission lines has recently been the subject of considerable interest [1]–[10]. These modes are usually undesirable since they result in increased attenuation of the signal, and may result in crosstalk with adjacent circuit components and other spurious effects, including interference with bound modes that also propagate on the line [3]. Of particular interest is the existence of leaky *dominant* modes on the structure [1]–[5]. A dominant leaky mode (as opposed to a leaky *higher order* mode, investigated in [6] and [7]) is one that has a current

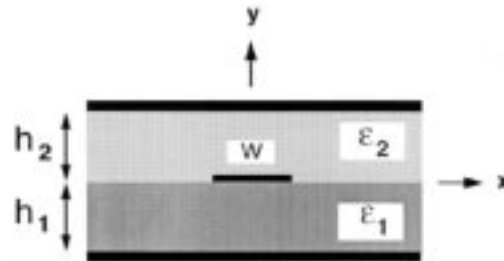


Fig. 1. Two-layered stripline structure.

distribution on the conducting strip that closely resembles that of a quasi-TEM mode of propagation. Therefore, such a leaky mode will typically be excited quite strongly by a customary feed. Leaky dominant modes have been found on multilayer stripline structures [2]–[5], coplanar waveguide and slotline [8], coplanar strips [9], microstrip lines with an anisotropic substrate [10], and recently on microstrip lines with an isotropic substrate [11].

Although the propagation properties of leaky modes on printed-circuit lines have been studied quite thoroughly in recent years, much less attention has been devoted to the interesting and practical issue of excitation of these modes by a practical source of finite size, such as a delta-gap feed on the conducting strip or a probe feed. The issue of excitation by a finite-size feed is an important one since it can be used to *define* the degree of physical meaning of the leaky mode [12]. In this paper, the *excitation* of leaky modes on printed-circuit lines by a *finite source* is investigated. Although the method is general, results will be presented for the two-layer stripline structure shown in Fig. 1. One advantage of this structure is that it allows considerable flexibility in controlling the phase constants of the leaky, bound, and parallel-plate modes, by selection of the dimensions and permittivities. The source may consist of an infinitesimal dipole located in proximity to the conducting strip or a delta-gap feed located on the strip.

The physical meaning of a leaky mode is defined here by the degree to which the fields of the leaky mode resemble the *complete* fields existing on the structure when excited by the *finite source*. Since the leaky mode is improper, the actual fields will not agree with the leaky-mode fields everywhere in space (this is well-known [13]). However, if the leaky mode is physically meaningful, the leaky-mode fields are expected to agree well with the actual fields within a limited angular region of space determined by the leakage angle [13], [14]. This angular region of space always includes the interface between the leaky structure and the exterior region into which

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radiation occurs. In particular, for a leaky printed-circuit line, the angular region of validity always includes the printed line itself. Therefore, if a leaky mode on a printed-circuit line is physically meaningful, the current of the leaky mode on the conducting strip that is excited by the source would be expected to agree well with the *complete* strip current due to the source excitation (agreement both in amplitude and phase). Therefore, we *define* the degree of physical meaning of a leaky mode on a printed-circuit line by the degree to which the strip current of the leaky mode agrees with the complete strip current. The excitation of the current on the conducting strip by a finite source thus provides a convenient way to investigate the physical meaning of a leaky mode on a printed-line structure.

In this paper, the calculation of the strip current due to the finite source excitation is performed by constructing a numerical Green's function, which gives the current induced on an *infinite* strip conductor due to the *finite* source. The calculated strip current (referred to here as the “complete” current) is numerically exact, under the assumption that the strip width is small (since a fixed transverse dependence of the longitudinal current is assumed, and the transverse current is neglected). The numerical Green's function is obtained by Fourier transforming which is transforming the source in the longitudinal (z)-direction. Thus, the problem is essentially reduced to one of calculating the strip current due to an infinite set of phased-line source excitations (a *one-dimensional* Green's function problem). The one-dimensional Green's function is in turn calculated from a spectral integration in the transverse wavenumber (k_x) plane, which is the same type of integration used to solve for the modal solutions on the guiding structure [15]–[19]. One of the properties of the k_x integration is that different choices are possible for the path of integration [2]. A real-axis path defines a modal solution that is bound in the transverse ($\pm x$) directions, while a path that detours around the poles of the background structure results in a solution that is improper in the transverse directions. It is shown here that the different choices of path in the transverse wavenumber plane give rise to *branch cuts* in the longitudinal wavenumber plane for the integration in k_z that determines the numerical Green's function. A careful consideration of these branch cuts provides much insight into the physical meaning of the leaky modes that are excited by the source, corresponding to the poles in the k_z plane.

The complete current is compared to the current of the leaky mode alone, defined from the residue contribution of the leaky-wave pole in the k_z integration of the numerical Green's function. The degree of physical meaning for the leaky mode is defined by the correlation between the complete and leaky-mode currents. The generalized pencil of functions (GPOF) method [20]–[25], which resolves the complete current into a set of exponential waves, is used to help quantify the correlation.

II. ANALYSIS

A. Formulation for Strip Current

Fig. 2 shows two possible excitations for a multilayer stripline structure: a unit-strength vertical electric dipole

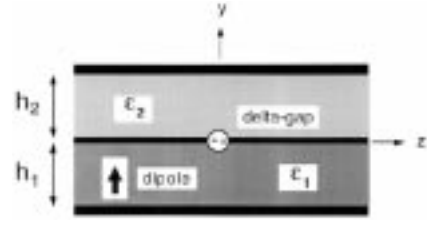


Fig. 2. VED and delta-gap excitations for the two-layered stripline.

(VED) and a delta-gap feed. The VED serves as a model for a probe feed. The conducting strip is assumed to be infinite in the $\pm z$ directions, and is perfectly conducting. Another assumption is that the strip width is sufficiently small that the transverse (x -directed) current may be neglected. The VED source is considered first.

The vertical dipole is represented as a planar sheet of vertical current at $y = y'$, having the form

$$J_y(x, z) = T(x)L(z), \quad (1)$$

For the dipole, $T(x) = \delta(x)$ and $L(z) = \delta(z)$, but the present derivation is general. The current source is then represented as

$$J_y(x, z) = T(x) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{L}(k_z) e^{-jk_z z} dk_z. \quad (2)$$

This equation decomposes the original source into an infinite set of phased-line source currents, each having the form

$$J_y^l(x, z) = A(k_z)T(x)e^{-jk_z z} \quad (3)$$

where

$$A(k_z) = \frac{1}{2\pi} \tilde{L}(k_z) dk_z.$$

The field E_z at y (the location of the strip) from an arbitrary planar sheet of vertical current $J_y(x, z)$ at y' can be written as

$$E_z(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{J}_y(k_x, k'_z) \cdot \tilde{G}_{zy}(k_x, k'_z; y, y') e^{-j(k_x x + k'_z z)} dk_x dk'_z \quad (4)$$

where $\tilde{G}_{zy}(k_x, k'_z; y, y')$ is the spectral Green's function for the field E_z due to a vertical dipole at $y = y'$ [the notation k'_z is used to avoid confusion with k_z appearing in (3)]. Applying (4) to the phased-line current in (3), and using the Fourier integral representation of the delta function gives the field from the phased-line source as

$$E_z^l(x, y, z) = A(k_z) e^{-jk_z z} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{T}(k_x) \cdot \tilde{G}_{zy}(k_x, k_z; y, y') e^{-jk_x x} dk_x. \quad (5)$$

This field from each phased-line current acts as an incident field that induces a phased current on the strip, having the form

$$J_{sz}^p(x, z) = B(k_z)\eta(x)e^{-jk_z z} \quad (6)$$

where $B(k_z)$ is an amplitude function that depends on k_z , and the transverse current function $\eta(x)$ is taken as

$$\eta(x) = \begin{cases} \frac{1}{\pi \sqrt{\left(\frac{w}{2}\right)^2 - x^2}}, & w/h_{\min} < 2 \\ \frac{1}{w}, & w/h_{\min} \geq 2 \end{cases} \quad (7)$$

where $h_{\min} = \min(h_1, h_2)$. The same transverse current dependence is assumed for all k_z (valid for a narrow strip). Similar to (5), the field due to the phased strip current in (6) is

$$E_z^p(x, y, z) = B(k_z) e^{-jk_z z} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\eta}(k_x) \cdot \tilde{G}_{zz}(k_x, k_z; y, y) e^{-jk_x x} dk_x. \quad (8)$$

where

$$\tilde{\eta}(k_x) = \begin{cases} J_0(k_x w/2), & w/h_{\min} < 2 \\ \text{sinc}(k_x w/2), & w/h_{\min} \geq 2 \end{cases} \quad (9)$$

with J_0 the zeroth-order Bessel function and $\text{sinc}(x) \equiv \sin(x)/x$. The electric-field integral equation (EFIE) states that the field due to the strip current in (8) must cancel the field due to the line source in (5). The EFIE is enforced by multiplying both sides of the resulting equation by $\eta(x)$ and integrating over the strip width (Galerkin testing). The resulting equation yields the solution for the amplitude function as

$$B(k_z) = A(k_z) R_v(k_z) \quad (10)$$

where

$$R_v(k_z) = - \frac{\int_{-\infty}^{\infty} \tilde{T}(k_x) \tilde{\eta}(k_x) \tilde{G}_{zy}(k_x, k_z; y, y') dk_x}{\int_{-\infty}^{\infty} \tilde{\eta}^2(k_x) \tilde{G}_{zz}(k_x, k_z; y, y) dk_x}. \quad (11)$$

The complete current induced on the conducting strip due to the VED source is then

$$J_{sz}(x, z) = \eta(x) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{L}(k_z) R_v(k_z) e^{-jk_z z} dk_z. \quad (12)$$

For the particular case of the VED source ($L(z) = \delta(z)$), (12) reduces to

$$J_{sz}(x, z) = \eta(x) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} R_v(k_z) e^{-jk_z z} dk_z. \quad (13)$$

Equation (13) is the numerical Green's function for the exact current on the strip due to the VED. The function $R_v(k_z)$ is given by (11) with $\tilde{T}(k_x) = 1$.

The analysis for the case of a z -directed horizontal electric dipole (HED) source is very similar to that for the VED, and is omitted. The final result is the same as (11) with $\tilde{G}_{zy}(k_x, k_z; y, y')$ replaced with $\tilde{G}_{zz}(k_x, k_z; y, y')$.

The function $R_v(k_z)$ has poles in the k_z plane at the values of the propagation constants of the guided modes on the structure, either k_z^b for the bound mode or k_z^w for a leaky mode. This is because the denominator in (11) is precisely the same integral that appears in the solution of the propagation constant for the guided mode on the structure. The residue

contribution to the integral (13) at a pole gives, by definition, the current amplitude of the guided mode (either bound or leaky) and defines the excitation coefficient of the guided mode.

For the delta-gap feed, the analysis is somewhat different because the source is an impressed electric field on the surface of the conducting strip (the delta-gap field). The impressed electric field is represented as

$$E_z(x, z) = T(x) L_g(z). \quad (14)$$

The function $T(x)$ is taken as unity over the width of the strip. The longitudinal gap function $L_g(z)$ is $\delta(z)$ for an idealized delta-gap feed. However, in order to make the transform converge faster, $L_g(z)$ is taken as

$$L_g(z) = \frac{1}{d} \frac{e^{-(1/2)(z/d)^2}}{\sqrt{2\pi}} \quad (15)$$

where d is an effective gap width. This function has the transform

$$\tilde{L}_g(k_z) = e^{-(1/2)(k_z d)^2}. \quad (16)$$

The impressed electric field on the strip is then represented as

$$E_z(x, z) = T(x) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{L}_g(k_z) e^{-jk_z z} dk_z \quad (17)$$

which is a collection of phased sources of the form

$$E_z(x, z) = A(k_z) T(x) e^{-jk_z z} \quad (18)$$

where

$$A(k_z) = \frac{1}{2\pi} \tilde{L}_g(k_z) dk_z.$$

The current on the strip is represented as a collection of phased currents, as shown in (6), each producing the field shown in (8). The field in (8) produced by the phased current is equated with that of (18), and the resulting equation is enforced by integrating over the strip width after multiplying by $\eta(x)$. This results in

$$B(k_z) = A(k_z) R_g(k_z) \quad (19)$$

where

$$R_g(k_z) = 2\pi \frac{\int_{-w/2}^{w/2} \eta(x) T(x) dx}{\int_{-\infty}^{\infty} \tilde{\eta}^2(k_x) \tilde{G}_{zz}(k_x, k_z; y, y) dk_x}. \quad (20)$$

Using (7), and recalling that $T(x) = 1$ for the delta-gap case, the integral in the numerator of (20) is unity. Hence,

$$R_g(k_z) = \frac{2\pi}{\int_{-\infty}^{\infty} \tilde{\eta}^2(k_x) \tilde{G}_{zz}(k_x, k_z; y, y) dk_x}. \quad (21)$$

The total current on the strip is then

$$J_{sz}(x, z) = \eta(x) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{L}_g(k_z) R_g(k_z) e^{-jk_z z} dk_z. \quad (22)$$

Equation (22) is the numerical Green's function for the current on the strip when excited by a delta-gap feed.

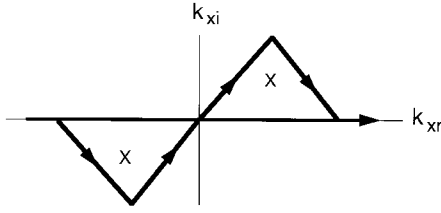


Fig. 3. Two possible paths of integration in the k_x plane. The real-axis path yields the bound-mode solution, while the path that detours around the poles yields the leaky-mode solution.

B. Discussion of Integration Paths

An important consideration in the evaluation of the numerical Green's function is the choice of path in the k_x plane for the evaluation of the function $R_v(k_z)$ in (11) or $R_g(k_z)$ in (21). There are poles in the k_x plane corresponding to the parallel-plate modes of the background structure, located at

$$k_x = k_{xp} = (k_{pp}^2 - k_z^2)^{1/2} \quad (23)$$

where k_{pp} is the propagation wavenumber of a parallel-plate mode. In most practical cases, only the fundamental TM_0 mode ($k_{pp} = k_{TM0}$) is above cutoff. The path of integration in the k_x plane may or may not either be chosen to detour around the poles. For example, if k_z is chosen in the fourth quadrant of the complex plane, the poles in the k_x plane will be in the first and third quadrants, as shown in Fig. 3 (illustrated for a single pair of poles corresponding to $k_{pp} = k_{TM0}$). There are two possible paths shown: the real axis path (which will yield a bound solution) and the one that detours around the poles (which will yield an improper solution). Therefore, the function $R(k_z)$ (which denotes either $R_v(k_z)$ or $R_g(k_z)$) is a multivalued function, which implies the existence of branch cuts in the k_z plane in order to restrict the function $R(k_z)$ to being single valued. These branch cuts play a crucial role in providing insight into the physical meaning of the leaky modes excited on the structure.

To examine the nature of the branch cuts and establish where the branch points are, it is helpful to note that (23) maps the real axis of the k_x plane to that part of the axes in the k_z plane labeled R in Fig. 4(a) (solid line). The imaginary axis in the k_x plane is mapped to the part of the real axis in the k_z plane that is labeled I (dashed line). The origin of the k_x plane is mapped to the point k_{TM0} in the k_z plane. As a point in the k_z plane moves around the point k_{TM0} , the poles in the k_x plane cross the real axis and then the imaginary axis to return to their original positions. This is illustrated in Fig. 4(b)–(d), which shows the pole locations corresponding to the points labeled 1–3 in Fig. 4(a). Note that the path of integration is continuously deformed as the poles move, so that the integration is a continuous function of k_z . At point 3, one complete trip around the point k_{TM0} has been made. The path in the k_x plane has changed from the real axis (point 1) to a path that detours around the poles (the path that is used to obtain the leaky-mode solutions). This demonstrates that the point k_{TM0} is a *branch point* in the k_z plane. The

same conclusion holds for the point $-k_{TM0}$. Fig. 4(e) and (f) correspond to points 4 and 5 in Fig. 4(a), and shows the evolution of the path as the point in the k_z plane circles the branch point at k_{TM0} once again. As the point ends up at the starting position (point 5), the path in Fig. 4(f) detours around each pole in the k_x plane twice in opposite directions. This is equivalent to the real-axis path of Fig. 4(b). Hence, it is concluded that the branch cut corresponds to a *two-sheeted* Riemann surface (as does a square-root type of branch point).

The important observation that *poles of the background structure* give rise to *branch points in the longitudinal wavenumber plane*, which is key to the discussion below on the physical meaning of the leaky modes, was recognized originally by Mesa and Marqués [19] and by Nyquist and Infante [26]. The proof given here, based on considerations of the path of integration in the k_x plane, provides additional insights and complements the discussions given in [19] and [26].

Branch points will occur in the k_z plane at all points $k_z = k_{pp}$. In most cases, only one mode (TM_0), or at most two (TM_0 and TE_1), are above cutoff. All of the modes below cutoff correspond to branch points on the imaginary axis of the k_z plane (the propagation constant of a parallel-plate mode below cutoff is purely imaginary). Accounting for all possible modes, there are an infinite number of sheets, two from each branch point. To completely specify where a point in the k_z plane is, it is necessary to indicate which sheet (top or bottom) the point is on for each of the branch points. Fortunately, only a few of the possible combinations correspond to physically meaningful leaky-mode pole locations, as will be explained shortly.

The exact shape of the *branch cuts* is arbitrary. However, a convenient choice is the Sommerfeld branch cut, in analogy with the same shape of branch cut that is commonly used when dealing with the wavenumber mapping shown in (23). The Sommerfeld choice of branch cuts is shown in Fig. 5(a) for the case of one mode above cutoff, and in Fig. 5(b) for the case of two modes above cutoff. Also shown in these figures are the poles corresponding to the bound (k_z^b) and leaky (k_z^{lw}) modes that can propagate on the stripline structure. A convenient property of the Sommerfeld branch cut is that all points on one of the sheets (denoted as the top sheet) of the k_z plane correspond to paths in the k_x plane that do not detour around the poles in the k_x plane—that is, the path is the *real axis*. Points on the *bottom sheet* correspond to paths that *detour around the poles* in the k_x plane (such a path is equivalent to the real axis path plus the residue contribution from the captured poles). This is in analogy with the usual property of the Sommerfeld branch cuts for radiation problems, where the top sheet is “proper” and the bottom sheet is “improper.”

The branch cuts in the k_z plane provide insight into the physical meaning of a leaky mode that is excited by the source. The path of integration in the k_z plane in (13) or (22) is along the real axis, except that the path detours around the bound-mode poles that lie on the real axis (above the pole on the positive real axis, below the one on the negative real axis). The path stays on the top sheet of all branch points. (This path results in a total field that is bounded in space, which must be the case for the field from a finite-source excitation). If a leaky-

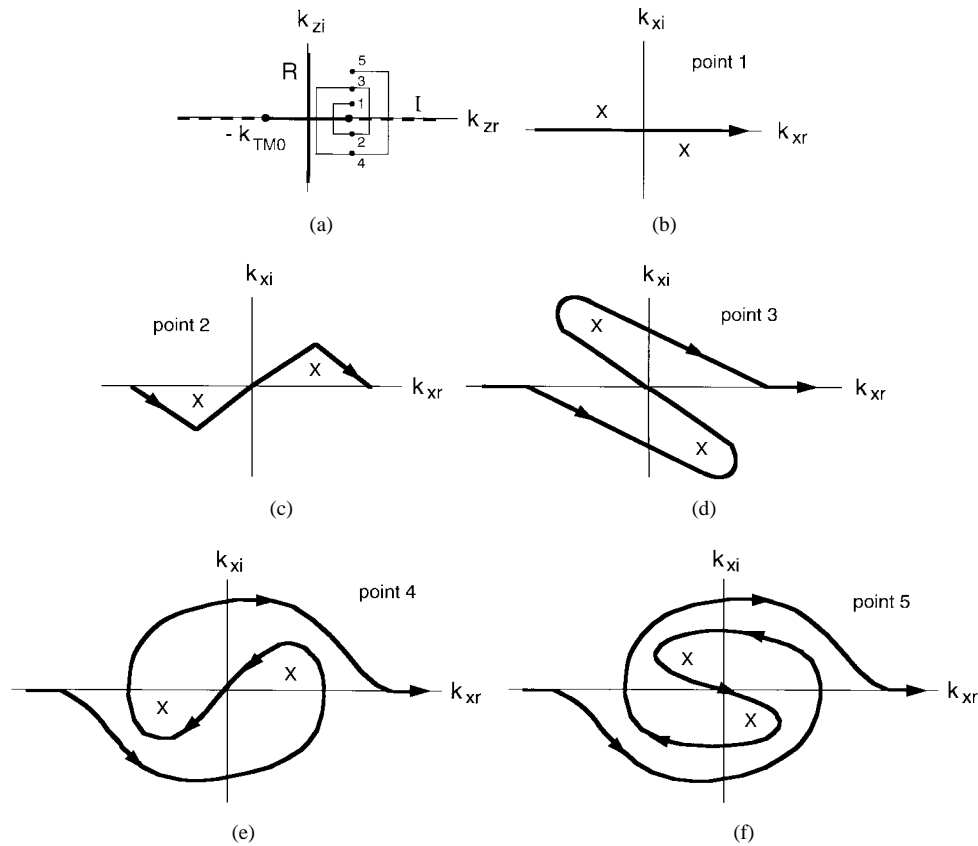


Fig. 4. (a) The k_z plane, showing the mapping of the real k_x axis as a solid line (R) and the imaginary k_x axis as a dashed line (I). The line R is the entire imaginary axis of the k_z plane, while I is the real axis with the part between the branch points excluded. Also shown is a path that encircles the point k_{TM0} twice, with various positions labeled. (b)–(f) Show paths in the k_x plane, corresponding to the various values of k_z labeled in part (a).

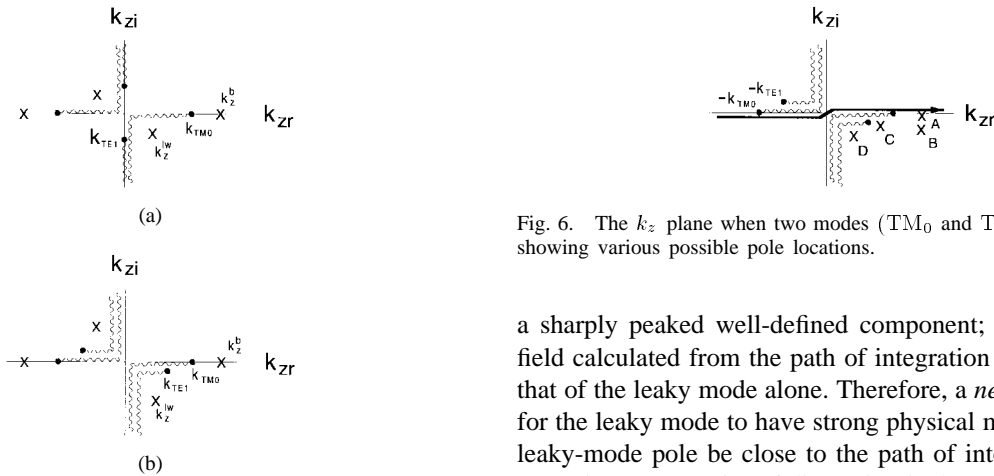


Fig. 5. (a) Sommerfeld branch cuts when only one mode (TM_0) is above cutoff. (b) Sommerfeld branch cuts when two modes (TM_0 and TE_1) are above cutoff.

mode pole is close to the integration path in the k_z plane, and the residue of the pole is not too small, then the pole will make a strong contribution to the path integration. This contribution will result in a strip current that closely resembles the current of the leaky mode, i.e., the contribution from the residue of the leaky-wave pole k_z^{lw} . If the pole is further from the path, its contribution will be blurred out, and the integrand will not have

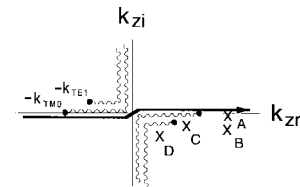


Fig. 6. The k_z plane when two modes (TM_0 and TE_1) are above cutoff, showing various possible pole locations.

a sharply peaked well-defined component; consequently, the field calculated from the path of integration will not resemble that of the leaky mode alone. Therefore, a *necessary* condition for the leaky mode to have strong physical meaning is that the leaky-mode pole be close to the path of integration.

An important point of discussion is the word “close.” Here, the term “close” means close in the *Riemann surface* sense, not in the geometrical sense. To illustrate this, consider several possible pole locations in the k_z plane of Fig. 5(b), shown in Fig. 6 as points A, B, C, and D (only poles in the right half of the complex plane are shown for simplicity). Point A is assumed to be on the top sheet of all branch points. Point A corresponds to the location of the bound-mode pole, which is on the real axis and has $k_{zr} > k_{TM0}$, while points B, C, and D are possible locations of a leaky-mode pole. Clearly, point A is close to the path (and this agrees with the well-known fact that the bound mode is always physically meaningful).

Point B has $k_{zr} > k_{\text{TM}_0}$, but is assumed to reside on the bottom sheet of the TM_0 branch point, and the top sheet of all others (point B would correspond to the path shown in Fig. 3, which detours around the TM_0 parallel-plate poles). Since point B is on the opposite sheet to the one the path is on, it is “far” from the path, even though it may have coordinates (k_{zr}, k_{zi}) that are *geometrically* close to the path. Point C has $k_{\text{TE}_1} < k_{zr} < k_{\text{TM}_0}$, and is on the bottom sheet of the TM_0 branch point and the top sheet of all others. This point is close to the path since the path is reached by smoothly moving upward from the point on the Riemann surface, passing from the bottom sheet to the top sheet while continuously changing the value of the function. A point at position C , but on the top sheet of the TM_0 branch point, would be far from the path in the Riemann sense.

Point D has $k_z < k_{\text{TE}_0}$, and is located on the bottom sheet of both the TM_0 and TE_1 branch points. This point is also close to the path since the path is reached by smoothly crossing both branch cuts (moving continuously on the Riemann surface to the sheet that is proper for all branch points).

For a leaky-mode pole at points C or D , any other location on different sheets other than those mentioned above would not be close to the path. A concise way to summarize this necessary condition for physical meaning is that provided by the “path consistency condition.” This condition was discussed in [2] (where it was termed the “condition of leakage”) as a speculative criterion for when leaky modes may be physically meaningful. It relates to the choice of path shown in Fig. 3. This criterion states that in order for a leaky mode to have well-established physical meaning, the value of the phase constant $\beta = \text{Re}(k_z^{lw})$ must be *consistent* with the choice of path used to obtain the leaky mode. The word “consistent” means that the path must detour around (capture the residues from) only those poles for which $\beta < k_{pp}$ and no others. (Of course, the value of the propagation constant is not known until after the numerical solution is obtained. The numerical solution used to obtain the propagation constant involves the path of integration in the k_x plane and, hence, it is not usually known if the solution will be consistent with the path in advance.) This speculative condition was discussed in [2] by using physical reasoning (without mathematical justification). Physically, if a leaky mode has $\beta > k_{pp}$ for a certain parallel-plate mode, conventional reasoning dictates that the leaky mode should not radiate into this parallel-plate mode. This physical reasoning is discussed in [1]. It is seen that the “path consistency condition” is precisely the same condition as requires the leaky-mode pole to be close to the path of integration in the Riemann sense. For example, if $k_{\text{TE}_1} < k_{zr} < k_{\text{TM}_0}$ (point C in Fig. 6), the pole is close to the path if it is on the bottom sheet of only the TM_0 branch point, and not the TE_1 branch point. This corresponds to a path of integration in the k_x plane that detours around only the TM_0 poles, not the TE_1 poles.

Although the previous conclusions have been illustrated for the case of one or two parallel-plate modes above cutoff, the preceding argument can easily be generalized to any number of modes above cutoff. The conclusion is that the path consistency condition is a necessary condition for a leaky mode to have well-established physical meaning.

III. RESULTS

In this section, the complete current on the conducting strip is compared with the current of the leaky mode in order to illustrate the determination of physical meaning of the leaky mode. The complete current on the strip is calculated from (13) for the VED or (22) for the delta-gap source (which assumes a single basis function of current for the transverse variation). The current of the leaky mode is obtained by calculating the residue contribution from the corresponding integrals, which *defines* the excitation coefficient of the leaky mode. (The excitation coefficient of the bound mode is also calculated in this way, although the focus of the results will be on the leaky mode). Results have also been obtained using multiple basis functions for the longitudinal current and using basis functions for the transverse (x -directed) current. These results have only shown very small differences with the results obtained using a single basis function, since the strip width is small compared to a wavelength for all of the results shown here.

To help quantify the comparison, the GPOF method [20]–[25] is used to approximate the complete strip current with a set of exponential waves. The development of the GPOF method [20] and its recent application to printed-circuit structures by Sarkar *et al.* [25] provides an effective tool for the characterization of the current on a printed-circuit line. In this study, the amplitude and propagation constants of the GPOF approximation are compared with the theoretical amplitudes (excitation coefficients) and propagation constants of the bound and leaky modes, and this comparison is used to explore the physical meaning of the leaky modes.

The dispersion curves showing the normalized phase constants for the bound, leaky, and TM_0 modes of the two-layered structure in Fig. 1 are shown in Fig. 7(a). Fig. 7(b) shows the normalized attenuation constant of the leaky mode. The structure has been designed with a wide strip and with the permittivity of the bottom layer much larger than that of the top layer (which is air). This results in a large separation between the dispersion curves for the three different solutions in Fig. 7(a), which makes the results easier to interpret. The “spectral gap” [27] begins at about 1.25 GHz. Below this frequency, $\beta > k_{\text{TM}_0}$ and the leaky-mode solution thus violates the “path consistency condition,” which means that it is not expected to have much physical meaning. Below 0.5 GHz, the leaky solution does not exist and, instead, a pair of improper real solutions are found [3], [5]. These improper real solutions have no physical significance and are not shown in Fig. 7(a).

Tables I and II first present results for the structure in Fig. 7 to verify the convergence of the GPOF method as the number of sample points, length of the sampling interval, and precision parameter are varied. (The precision parameter is a negative integer $(-m)$ that determines the degree of fitting. The GPOF routine picks the number of exponential waves to obtain a fitting that is, roughly speaking, accurate to 10^{-m} [20].) For these tables, a delta-gap feed is used, and the frequency is 10 GHz. The theoretical excitation coefficients and propagation wavenumbers of the bound and leaky modes, as well as the propagation wavenumber of the TM_0 mode, are

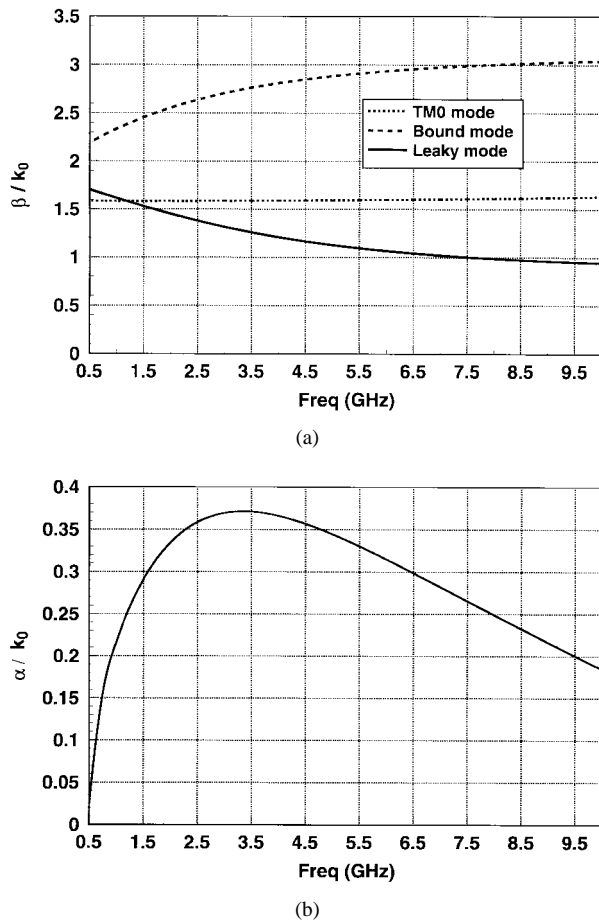


Fig. 7. (a) Dispersion plot showing the wavenumbers of the bound and leaky modes for the structure of Fig. 1. Also shown is the dispersion curve for the propagation wavenumber of the TM_0 parallel-plate mode. (b) The attenuation (leakage) constant for the leaky mode. $\epsilon_{r1} = 10.0$, $\epsilon_{r2} = 1.0$, $h_1 = 1.0$ mm, $h_2 = 0.5$ mm, $w = 10.0$ mm.

shown in Table I. The amplitude and propagation constants of the GPOF waves are shown in Table II for various number of sampling points N (the number of waves varies with the parameter $-m$, as this is picked by the routine). The sampling region starts at $L_1 = 1.0\lambda_0$ in all cases to avoid sampling too near the source (since the near-field current is not well approximated by only the bound and leaky modes).

Comparing Tables I and II, it is seen that the agreement between the first two GPOF waves and the bound- and leaky-mode current waves is always good, but improves as the number of sampling points, upper limit of the sampling interval (L_2), and precision increase. For 1000 sampling points and $L_2 = 9.0\lambda_0$, the agreement is excellent.

One interesting point in connection with Table II is the clear indication of a “residual wave” current on the strip. By definition, the residual wave is the difference between the complete (total) strip current and the current due to the leaky and bound modes. It is that part of the continuous spectrum current (complete current minus the current of the bound mode) that is not well approximated by the leaky-mode current. The continuous spectrum current is represented by an integral around the branch cuts in Fig. 5. Asymptotically, for large z , the continuous-spectrum current is dominated

TABLE I
THEORETICAL PROPAGATION CONSTANTS AND AMPLITUDES (REAL, IMAGINARY) FOR A DELTA-GAP SOURCE AT 10 GHz. THE STRUCTURE IS THE SAME AS IN FIG. 7

MODE	k_{z0}/k_0	AMPLITUDE
TM0	(1.63075, 0)	NA
bound	(3.04248, 0)	(0.01512, 0)
leaky	(0.94444, -0.18471)	(0.01325, -0.00287)

TABLE II
PROPAGATION CONSTANTS AND AMPLITUDES (REAL, IMAGINARY) FROM THE GPOF METHOD FOR A DELTA-GAP SOURCE AT 10 GHz. THE STRUCTURE IS THE SAME AS IN FIG. 7

N	L_2/λ_0	m	k_{z0}/k_0	AMPLITUDE
100	2.0	-2	(3.04502, -0.00273)	(0.01534, 0.0004)
			(0.92311, -0.18535)	(0.01142, -0.0050)
500	5.0	-3	(3.04241, -0.000017)	(0.01511, 0.000003)
			(0.94343, -0.18490)	(0.01281, -0.0029)
			(1.63487, -0.10258)	(-0.00022, 0.00026)
1000	9.0	-4	(3.04241, 0.000013)	(0.01513, 0.000005)
			(0.94441, -0.18476)	(0.01352, -0.002715)
			(1.63160, -0.19947)	(-0.00065, -0.00027)
			(1.6314, -0.037262)	(-0.000070, 0.000094)

by the residual-wave current since the leaky-mode decays exponentially. Also, for large z , the branch-cut integrals are dominated by the branch-point contributions. Since the TM_0 mode is the only one above cutoff in this example, the residual-wave field should, therefore, be dominated by the branch-point contribution at the TM_0 branch point. This observation was originally pointed out and discussed in [26] in connection with higher order leaky modes excited on stripline structures. It explains why the residual GPOF waves (the ones in addition to the first two) have a propagation constant that is very close to that of the TM_0 parallel-plate mode. The residual-wave current does not decay exponentially, as the leaky-mode current does, so it often takes more than one GPOF wave to accurately model the residual wave.

Table III shows a comparison between the theoretical excitation coefficients and the GPOF fit versus frequency, to study how the physical meaning of the leaky mode changes with frequency. The structure is the same as that used in Table II, except that the strip is a little narrower (7 mm instead of 10 mm). The four GPOF waves with the largest amplitudes are shown for each frequency. The GPOF waves have been ordered so that the first one corresponds to the bound mode, while the second one corresponds to the leaky mode. Table III is shown in two parts, where (a) is for frequencies above 1.5 GHz, and (b) is for frequencies below 1.5 GHz (in the spectral-gap region). For all frequencies, the agreement between the theoretical values and the GPOF results is excellent for the bound mode, as expected (since this mode always has complete physical meaning). For the leaky mode, the agreement with the second GPOF wave is quite good at 10 GHz, and becomes progressively worse as the frequency

TABLE III

THEORETICAL PROPAGATION CONSTANTS AND AMPLITUDES (REAL, IMAGINARY) DETERMINED FROM THE POLE LOCATIONS AND THE RESIDUES OF THE POLES IN THE k_z PLANE, AND THOSE DETERMINED NUMERICALLY BY THE GPOF METHOD FOR A DELTA-GAP SOURCE. THE STRUCTURE IS THE SAME AS IN TABLES I AND II, EXCEPT FOR A DIFFERENT STRIP WIDTH

F (GHz)			k_{z0} / k_0	AMPLITUDE
10	Theory	Bound Leaky TM_0	(2.9827,0) (1.01223,-0.27386) (1.6307,0)	(0.01156,0) (0.00698,-0.00178)
10	GPOF		(2.9838,-0.00003) (1.0152,-0.27761) (1.6175,-0.1265)	(0.01154,0.000016) (0.00692,-0.00212) (-0.00016,0.000319)
8	Theory	Bound Leaky TM_0	(2.9277,0) (1.0800,-0.3283) (1.6119,0)	(0.01206,0) (0.00761,-0.00178)
8	GPOF		(2.9275,-0.00004) (1.0893,-0.3221) (1.6022,-0.1243)	(0.01204,0.000017) (0.00803,-0.00203) (-0.000267,0.000416)
6	Theory	Bound Leaky TM_0	(2.8393,0) (1.1821,-0.36192) (1.5980,0)	(0.012816,0) (0.0084,-0.000147)
6	GPOF		(2.8392,0.000014) (1.1952,-0.3358) (1.56846,-0.12211)	(0.012787,0.000013) (0.0083,0.000232) (-0.000308,0.000738)
4	Theory	Bound Leaky TM_0	(2.6936,0) (1.3336,-0.36879) (1.5885,0)	(0.01393,0) (0.00949,-0.000459)
4	GPOF		(2.6935,0.000059) (1.31157,-0.34938) (1.57192,-0.12323)	(0.01390,0.000013) (0.00789,0.0011) (-0.0006761,0.0009944)
2	Theory	Bound Leaky TM_0	(2.4453,0) (1.5441,-0.28270) (1.5830,0)	(0.015581,0) (0.01035,0.00266)
2	GPOF		(2.4453,0.000069) (1.43760,-0.2338) (1.5543,-0.05356)	(0.015536,0.000015) (0.00241,0.00027) (-0.000025,0.0010)

(a)

F (GHz)			k_{z0} / k_0	AMPLITUDE
1	Theory	Bound Leaky TM_0	(2.2618,0) (1.66746,-0.14244) (1.5816,0)	(0.0167155,0) (0.010367,0.009650)
1	GPOF		(2.2620,-0.00022) (1.42276,-0.5782) (1.522,-0.09656)	(0.016705,0.000068) (0.00027,-0.001167) (-0.00172,0.00137)
0.75	Theory	Bound Leaky TM_0	(2.20785,0) (1.6995,-0.0559) (1.5814,0)	(0.017081,0) (0.01028,0.02744)
0.75	GPOF		(2.20789,-0.00026) (1.39027,-0.59179) (1.52807,-0.08489)	(0.017082,0.000051) (-0.000026,-0.000763) (0.00152,0.001039)

(b)

decreases. This is because the leaky mode is approaching the spectral gap. (The loss of physical meaning as the spectral gap is approached has been previously studied for a simpler structure, consisting of a line source inside a dielectric-layer leaky-wave antenna in [12].) Even at 4 GHz, where the leaky solution is not yet in the spectral gap, the leaky mode has already begun to lose physical meaning, as can be seen by comparing the theoretical amplitude of the leaky mode and the amplitude of the second GPOF wave. At 2 GHz, close to the spectral-gap boundary, there is no agreement between the theoretical leaky-mode amplitude and the second GPOF wave amplitude, although there is still a somewhat reasonable agreement between the propagation constants. Below 2 GHz, it is not even possible to determine which of the GPOF waves matches with the leaky mode since there is no agreement at all in either amplitude or propagation constant. In this frequency range, the leaky mode has lost all physical meaning.

TABLE IV

THEORETICAL AND GPOF PROPAGATION CONSTANTS AND AMPLITUDES (REAL, IMAGINARY) FOR DELTA-GAP AND VED SOURCES AT 8.0 GHz. THE STRUCTURE IS THE SAME AS IN TABLE III

SOURCE			k_{z0} / k_0	AMPLITUDE
delta-gap	Theory	bound leaky TM_0	(2.9277, 0) (1.0800, -0.32283) (1.6119, 0)	(0.01206, 0) (0.00761, -0.00178)
delta-gap	GPOF		(2.9275, 0.000015) (1.0828, -0.32201) (1.5974, -0.23874) (1.6110, -0.04738)	(0.01204, 0.000017) (0.00803, -0.00203) (-0.000267, 0.000416)
VED	Theory	bound leaky TM_0	(2.9277, 0) (1.0800, -0.32283) (1.6119, 0)	(-0.41087, 0) (-6.4156, -1.5993)
VED	GPOF		(2.9275, -0.000067) (1.0809, -0.32338) (1.6255, -0.21862) (1.6152, -0.04441)	(-0.40950, -0.00035) (-6.3082, -1.6027) (0.19026, -0.00197) (0.03897, -0.02337)

Although the quantitative comparison between the complete and the leaky-mode current will depend in part on the type of source and the specific structure, it is expected that the general conclusion provided by the results of Table III is still valid, and that the physical meaning of a leaky mode is gradually lost as the leaky mode enters the spectral-gap region. However, the rate at which the physical meaning is lost will, in general, depend on the type of structure.

Table IV shows a comparison between results obtained from a delta-gap feed and a VED feed for the same structure as in Table III at a frequency of 8.0 GHz. The VED source is in the upper layer (air region), halfway between the strip and upper ground plane. The four GPOF waves with the largest amplitudes are shown for each excitation. The VED source excites both the bound and leaky modes more strongly than the delta-gap feed does. However, perhaps the most interesting point is that the VED feed excites the leaky mode much more strongly than the bound mode, whereas the delta-gap feed excites both modes somewhat equally. This can be explained by the observation that the leaky-mode field is primarily confined to the upper layer, whereas the field of the bound mode is primarily confined to the lower layer. (Due to the wide strip, the leaky and bound modes are both quasi-TEM parallel-plate modes, existing in the upper and lower regions, respectively.) The vertical dipole is in the top layer, thus it primarily excites the leaky mode. The delta-gap feed “sees” both regions and, thus, excites both modes more equally.

IV. CONCLUSIONS

A quasi-analytical method for calculating the current on the conducting strip of a multilayered stripline structure when excited by a finite source has been presented. The formulation has been presented for a VED in proximity to the strip or a delta-gap feed on the strip. This method constructs a numerical Green’s function for the current on the strip due to the source by using a Fourier transform of the source excitation to decompose it into a set of phased sources that are infinite in the longitudinal direction. A Green’s function for a line-source

type of excitation (one-dimensional Green's function) is then used to calculate the solution due to each phased source. The numerical Green's function is used to calculate the complete current on the strip that is produced by the finite source and also the current due to only the bound or leaky modes that are excited by the source. The currents of the bound and leaky modes are conveniently calculated from the numerical Green's function by taking the residue contribution from the corresponding poles in the longitudinal wavenumber plane.

Since different paths of integration are possible for the calculation of the one-dimensional Green's function, *branch cuts* appear in the longitudinal wavenumber plane of integration for the calculation of the numerical Green's function. A careful examination of these branch cuts provides much insight into the physical meaning of the leaky modes that are excited on the structure. The physical meaning of a leaky mode is defined here by the degree of correlation between the complete current on the strip and that due to the leaky mode alone. It is concluded that the "path consistency condition," previously introduced as a speculative condition for the physical meaning of a leaky mode is indeed a correct *necessary* condition for a leaky mode to have well-established physical meaning.

Numerical results are presented that compare the currents of the bound and leaky modes with the complete current on the strip. The GPOF method is used to help quantify this comparison. It is established that a practical source may indeed excite a leaky mode with an appreciable amplitude, and that the physical meaning of a leaky mode does depend on the phase constant of the mode. As the "spectral-gap" is approached where the path consistency condition is violated, the leaky mode begins to lose physical meaning.

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